THE GUESSING METHOD

Example: \( 1x^2 + 6 = 0 \)

The first term is \( 1x^2 \), the middle term is \( +5x \), and the last term is \( +6 \).

Guess at the factors of the first term: \( x \cdot x \).

Guess at the factors of the last term: \( 2 \cdot 3 \).

Now write the factors for the last terms like this: \((x \ 2)(x \ 3)\).

From the FOIL, you want the "O" and the "I" to add/subtract [depending on the sign of the last term: \( + \) add and \( - \) subtract] to equal the middle term.

\[
\begin{align*}
+ 2x \\
+ 3x \\
\hline
5x \text{ (middle term)}
\end{align*}
\]

therefore :

\[ (x + 2)(x + 3) = 0 \]

\[ x + 2 = 0 \text{ or } x + 3 = 0 \]

\[ x = -2 \text{ or } x = -3 \]

Guessing Method Examples :

1. \( x^2 + 4x + 3 = 0 \)

\[ (x \ 1)(x \ 3) \]

\[ + 3x \]

\[ + 1x \]

\[ = 4x \text{ (middle term)} \]

\[ (x + 3)(x + 1) = 0 \]

\[ x + 3 = 0 \text{ or } x + 1 = 0 \]

\[ x = -3 \text{ or } x = -1 \]

2. \( a^2 + 8a - 20 = 0 \)

We have three choices :

\[
\begin{array}{ccc}
(a \ 1)(a \ 20) & (a \ 2)(a \ 10) & (a \ 4)(a \ 5) \\
20a & 10a & 5a \\
1a & 2a & 4a \\
\hline
19a & 8a & 1a
\end{array}
\]

Notice that we are looking for the difference (the last term above is negative "-20").

\[ + 8a \text{ is what we need } (+10a - 2a = +8a). \]

\[ (a - 2)(a + 10) = 0 \]

\[ a - 2 = 0 \text{ or } a + 10 = 0 \]

\[ a = 2 \text{ or } a = -10 \]
3. \( x^2 - 2x - 24 \)

Now we have four choices:

\[
\begin{array}{cccc}
(x & 1)(x & 24) & (x & 2)(x & 12) & (x & 3)(x & 8) & (x & 4)(x & 6) \\
24x & 12x & 8x & 6x \\
1x & 2x & 3x & 4x \\
23x & 10x & 5x & 2x \\
\end{array}
\]

Notice that we are looking for the difference (the last term above is negative "-24").

-2x is what we need (-6x + 4x = -2x), so:

\[(x - 6)(x + 4) = 0\]

\[x - 6 = 0 \text{ or } x + 4 = 0\]

\[x = 6 \text{ or } x = -4\]

4. \( 3x^2 - 6x - 72 = 0 \)

Look at all the combinations we have:

\[
\begin{array}{cccccc}
(3x & 1)(1x & 72) & (3x & 2)(1x & 36) & (3x & 3)(1x & 24) & (3x & 4)(1x & 18) & (3x & 6)(1x & 12) & (3x & 8)(1x & 9) \\
(1x & 1)(3x & 72) & (1x & 2)(3x & 36) & (1x & 3)(3x & 24) & (1x & 4)(3x & 18) & (1x & 6)(3x & 12) & (1x & 8)(3x & 9) \\
\end{array}
\]

Unfortunately, we have to try them all. Fortunately, we can simplify the equation:

\[
3(x^2 - 2x - 24) = 0
\]

\[
3(x - 6)(x + 4) = 0 \quad \text{(See problem #3.)}
\]

\[
x = 6 \text{ or } x = -4
\]