

Infinite Sets and Their Cardinalities

Contemporary Math (MAT-130)

Give the cardinal number of each set.

1. $\{27\}$
2. $\{a, 1, b, 2, c, 3\}$
3. $\{14, 14, 14\}$
4. $\{x \mid x \text{ is an integer that satisfies } x - 5 = 4\}$
5. $\{x \mid x \text{ is a whole number between 9 and 12}\}$
6. $\{x \mid x \text{ is a fraction}\}$
7. $\{h, i, j, \dots, m\}$
8. $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
9. \emptyset
10. $\{x \mid x \text{ is an irrational number}\}$
11. $\{\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \dots, \frac{18}{20}\}$
12. $\{\text{Monday, Tuesday, } \dots, \text{Sunday}\}$
13. $\{2, 4, 6, 8, 10, \dots\}$
14. $\{p, a, n, a, m, a\}$
15. The set of real numbers

Place each pair of sets into a one-to-one correspondence.

16. $\{a, b, c\}$ and $\{1, 2, 3\}$
17. $\{\text{Monday, Tuesday, Wednesday}\}$ and $\{\text{Jan, Feb, Mar, Apr}\}$
18. $\{z, e, n, o\}$ and $\{c, o, l, o, r\}$
19. $\{\text{Spring, Summer, Fall, Winter}\}$ and $\{\text{Green, Orange, Brown, White}\}$

Determine whether each pair of sets is equal, equivalent, both, or neither.

20. $\{1, 2, 3\}$ and $\{2, 1, 3\}$
21. $\{A, B, C\}$ and $\{a, b, c\}$
22. $\{5, 6, 7, 8\}$ and $\{8, 7, 6\}$
23. $\{51, 2\}$ and $\{5, 12\}$

Show that each set has a cardinal number \aleph_0 by setting up a one-to-one correspondence between the given set and the set of counting numbers.

24. The set of negative integers
25. $\{1, 3, 5, 7, 9, 11, \dots\}$

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Solutions:

1. 1

2. 6

3. 1

4. 1

5. 2

6. \aleph_0

7. 6

8. \aleph_0

9. 0

10. c

11. 18

12. 7

13. \aleph_0

14. 4

15. c

16.

$a \rightarrow 1$
$b \rightarrow 2$
$c \rightarrow 3$

17. Not possible

18.

$z \rightarrow c$
$e \rightarrow o$
$n \rightarrow l$
$o \rightarrow r$

19.

<i>Spring</i> \rightarrow <i>Green</i>
<i>Summer</i> \rightarrow <i>Orange</i>
<i>Fall</i> \rightarrow <i>Brown</i>
<i>Winter</i> \rightarrow <i>White</i>

20. Both

21. Equivalent

22. Neither

23. Equivalent

24.

$-1 \rightarrow 1$
$-2 \rightarrow 2$
$-3 \rightarrow 3$
\vdots
$-n \rightarrow n$

25.

$1 \rightarrow 1$
$3 \rightarrow 2$
$5 \rightarrow 3$
\vdots
$2n - 1 \rightarrow n$