# PreCalculus(MAT-180) Cumulative Review

1) Simplify each expression.

a. $(2x^3-1)^3\left(\frac{4}{3}\right)(x^3-4)^{1/3}(3x^2)+(x^3-4)^{4/3}(3)(2x^3-1)^2(6x^2)$

b. $\frac{(3x+5)^{2/3}(-3)(x+4)^{-4}-(x+4)^{-3}\left(\frac{2}{3}\right)(3x+5)^{-1/3}(3)}{(3x+5)^{4/3}}$

c. $\left(\frac{-1}{2}\right)(5x^2+3)^{-3/2}(2x)(7x+2)^{3/2}+(5x^2+3)^{-1/2}\left(\frac{3}{2}\right)(7x+2)^{1/2}(7)$

2) Solve each inequality. Express the solution set using interval notation.

a. $\frac{2}{x-1}\leq 3x+2$

b. $x^2+2x>15$

c. $4x^2+8x<5$

d. $\frac{6}{x-3}<\frac{3}{x+1}$

e. $-2\leq 4-6x<22$

3) Given $f(x)=2x^2+7x+5,\ find$:

a. $f(0)$

b. $f(-2)$

c. $f\left(\sqrt{y}\right)$

d. $f(2t)$

e. $f(a+3)$

f. $f(-x)$

4) Find the difference quotient $\frac{f(x+h)-f(x)}{h}$. Simplify your answer.

a. $f(x)=2x^2-3x+1$

b. $f(x)=\frac{3}{x}$

c. $f(x)=\sqrt{x-5}$

d. $f(x)=\frac{x+7}{3x}$

5) State the domain of each function.

a. $f(x)=\frac{x+5}{x^2-2x-35}$

b. $f(x)=\sqrt{16-x^2}$

c. $f(x)=\frac{x+5}{\sqrt{x^2-9}}$

d. $f(x)=\sqrt[4]{\frac{x-9}{x+2}}$

e. $f(x)=\frac{\sqrt{x}}{\sqrt{16-x^2}}$

6) Solve for x: $f(x)=\frac{7x^3-21x^2-70x}{7x}=0$

7) Graph: $f(x) = \begin{cases} x^2 + 2, & x < 0 \\ \sqrt{x+4}, & x \geq 0 \end{cases}$

8) Express $|x + 4|$ $as\ a\ piecewise - defined\ function.$

9) Graph each function. Identify any transformations from the parent function.
   a. $f(x) = 3|x - 2|$
   b. $f(x) = -(x + 5)^2 + 2$
   c. $f(x) = \frac{1}{3}(x + 2)^3 - 2$
   d. $f(x) = -\sqrt{5 - x} + 3$
   e. $f(x) = [\![x]\!] + 3$

10) Given $f(x) = x^2 - 5$ $and$ $g(x) = \sqrt{x+3},$ $find$:
   a. $(f + g)(6)$
   b. $(g - f)(-3)$
   c. $(f \bullet g)(5)$
   d. $\left(\frac{f}{g}\right)(x)$
   e. $The\ domain\ of\ \left(\frac{f}{g}\right)(x)$

11) Given $f(x) = \sqrt{x^2 + 2}$ $and$ $g(x) = x - 5,$ $find$:

   a. $(g \circ f)(5)$ b. $(f \circ g)(x)$ c. $(f \circ f)(x)$

12) Determine whether the function has an inverse. If so, find the inverse, and state the domain and range of $f$ and $f^{-1}$.

   a. $f(x) = \frac{x+7}{x-3}$
   b. $f(x) = \sqrt{x} - 1$
   c. $f(x) = \frac{x^2-4}{7}$
   d. $f(x) = |x + 3|$
   e. $f(x) = \frac{1}{x}$

13) Given $f(x) = x^3 - 7$ $and$ $g(x) = \sqrt[3]{x+7}$, show that f and g are inverse functions.

14) Write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.

   a. Vertex: $(3,2)$ Point: $(6,29)$
   b. Vertex: $(2,-3)$ $Point$: $(4,-15)$

15) Given the function: $f(x) = \frac{1}{5}x^2 - 4x + 15$

   a. Identify the vertex, axis of symmetry, and intercepts.
   b. Sketch the graph of $y = f(x)$.

16) Sketch the graph of each polynomial.

a. $f(x) = x^4 - 10x^2 + 9$
b. $f(x) = 3x^3 - 4x^2$
c. $f(x) = (x^2 - 9)(3x^2 - 4x - 4)$

17) Perform the operation and write the result in standard form $(a + bi)$.

a. $(3 + 2i)(7 + i)$
b. $\frac{2}{4-i} + \frac{4}{3+i}$

18) Simplify the complex number and write it in standard form $(a + bi)$.

$5i^4 + 3i^3 + 7i^2 - i$

19) Divide. Express your answer in the form: $quotient + \frac{remainder}{divisor}$.

a. $(x^4 - 4x^3 + x^2 + 10x - 8) \div (x^2 + 4)$
b. $(x^3 - 1) \div (x - 1)$

20) Find all the real zeros of each polynomial function and determine its multiplicity.

a. $f(x) = x^4 - 3x^2 - 4$
b. $f(x) = x^3 - 4x^2 + 4x$

21) Find all solutions of the polynomial equation.

a. $x^4 - x^3 - 6x^2 + 4x = -8$
b. $x^4 - 13x^2 = -36$
c. $x^4 - 16 = 0$

22) Use the given zero to find all the zeros of the function.

*Function:* $f(x) = x^3 - 2x^2 + x - 2$ *zero:* $i$

23) Find the simplest polynomial of degree three with real coefficients that has $\boldsymbol{zeros}$: $\mathbf{1}, \mathbf{3i}$

24) Find all the zeros of the polynomial and write the result as a product of linear factors.

a. $f(x) = x^3 + 2x^2 - 5x - 6$
b. $f(x) = x^4 - 3x^3 - 12x - 16$
c. $f(x) = x^4 - 39x^2 - 70x$

25) State the domain of the function. Identify any asymptotes and intercepts.

a. $f(x) = \frac{x-3}{x^2-9}$ b. $f(x) = \frac{x^2}{x-1}$ c. $f(x) = \frac{x^2+5x-2}{x^2+x+2}$

26) **Exponential Growth**: The world population *P* (in billions) was 6.4 in 2005. The exponential growth rate *k* was 1.2% per year.

a. Find the exponential growth function for the population ($P = P_0 e^{kt},\ k > 0$).
b. Find the world population in 2008.
c. When will the world population be 8 billion? 9 billion?
d. Find the time for the world population to double in size.

27) **Radioactive Decay**: Let Q represent a mass of carbon 14 (in grams) whose half-life is 5,730 years. The quantity of carbon 14 present after t years is given by the model $Q = Q_0 \left(\frac{1}{2}\right)^{\frac{t}{5730}}$

a. Determine the initial quantity.
b. Determine the quantity present after 5,730 years.
c. Determine the quantity present after 40,110 years.

28) **Radioactive Decay**: The initial quantity present of carbon 14 is 800 grams. If this substance has a half-life of 5,730 years,

a. Find the formula for the quantity present ($Q = Q_0 e^{kt}$).
b. Find the amount of substance present after 150 years.

29) Sketch the graph of each function. Find the domain and any asymptotes.

a. $f(x) = -\left(\frac{1}{2}\right)^{x-5} + 3$
b. $f(x) = 1 - log(x + 6)$
c. $f(x) = -log_3(7 - x) + 3$
d. $f(x) = \frac{1}{2}e^{-x}$
e. $f(x) = ln(2x)$
f. $f(x) = -3ln(x - 4)$

30) Condense each expression.

a. $\frac{1}{2}ln(4x - 3) - 3ln(x + 3) + 2ln(2x - 3) - 5\,ln\,x$
b. $\frac{1}{3}[ln\,x + 3\,ln\,x - 2ln(x + 5)]$
c. $\frac{1}{3}log\,27 - \frac{1}{2}log\,y - log\,z$

31) Expand each expression.

a. $log\left(\frac{\sqrt{2x^3}}{yz^2}\right)$
b. $ln\left(\sqrt[3]{\frac{(x-5)y}{3z}}\right)$
c. $ln\left(\frac{5x+z}{xy}\right)^3$

32) Solve for x.

a. $x = log_5 25$

b. $x = log_8 2$

c. $2 = log\, x$

d. $2^x = 8$

e. $3^2(3^x) = \frac{1}{27}$

f. $3(2^{3x-7}) + 1 = 49$

g. $2^{3x-7} = 4^{x-3}$

h. $log_5 x - 2log_5 2 = 3$

i. $log_4(2x + 7) + log_4 x = 1$

j. $ln\, x^2 - ln\, 5 = ln\, 20$

k. $2\, ln\sqrt{x} + 2 = 2$

l. $ln\, x^4 + ln\, x^2 - ln\, x^3 - 2 = 7$

m. $ln\frac{1}{e} + x = 5$

n. $\frac{1}{3} ln\, e^{3x} - 3e^{ln\, x} = 12$

o. $e^{2x} - 3e^x - 18 = 0$

p. $e^{2x+6} = \left(\frac{1}{e}\right)^{-x^2-3x}$

33) **Compound Interest**: You own a credit card that charges an annual interest rate of 16%, compounded continuously. $(A = Pe^{rt})$

a. Given the initial balance of $10,000, find your balance after 7 years.
b. How much time will it take for your debt to double?

34) **Depreciation**: A car is worth $12,000. Five years ago the car was brand new and worth $25,000.

a. Find a linear model of depreciation $V = mt + b$, where $V$ is the value of the car after $t$ years.
b. Find an exponential model of depreciation $V = ae^{kt}$

35) Determine the **exact** values of the six trigonometric functions of the indicated angle.

a. $\theta = 2\pi$

b. $\theta = \frac{3\pi}{4}$

c. $\theta = \frac{2\pi}{3}$

d. $\theta = -\pi$

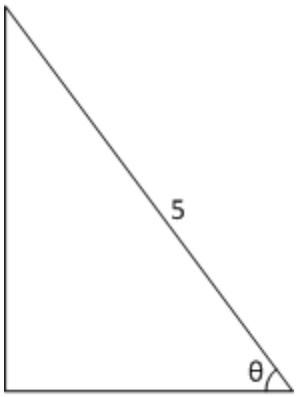
e. $\theta = -\frac{\pi}{3}$

f. $\theta = \frac{7\pi}{6}$

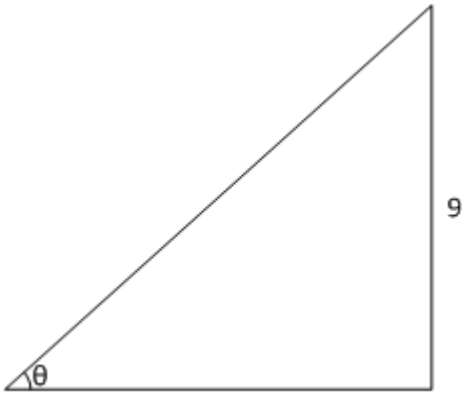
g. $\theta = -\frac{5\pi}{4}$ h. $\theta = -\frac{13\pi}{6}$ i. $\theta = -\frac{3\pi}{2}$

36) Find the values of the six trigonometric functions of the angle $\theta$ using the right triangles:

a.




b.




37) Find the remaining five trigonometric functions of $\theta$ satisfying the indicated condition.

a. $tan\,\theta = 1;\ \theta\ lies\ in\ quadrant\ III$
b. $cos\,\theta = -\frac{\sqrt{3}}{2};\ \theta\ lies\ in\ quadrat\ II$
c. $sec\,\theta = -\frac{5}{2};\ sin\,\theta > 0$
d. $cot\,\theta = \sqrt{3};\ cos\,\theta > 0$

38) Find the amplitude, period, and phase shift.

a. $y = 5\,sin\,2x$
b. $y = -\pi\,sin\left(\frac{\pi x}{4}\right)$
c. $y = -3\,cos(2x - \pi) + 1$
d. $y = -\,tan\left(x + \frac{\pi}{4}\right)$

39) Sketch the graph of each function. Include two full periods. Label any asymptotes and find the domain of the trigonometric function.

a. $y = 5 - sin\,2x$
b. $y = 3\,sin\,\pi x$
c. $y = -3\,sin(2x - \pi) + 1$
d. $y = 2\,cos\,2x - 2$
e. $3 + cos\left(\frac{\pi x}{4}\right)$
f. $y = tan\frac{1}{2}x$
g. $y = -tan\left(x - \frac{\pi}{3}\right)$
h. $y = 4\,cot\left(\frac{\pi x}{4}\right)$
i. $y = 2\,csc\left(x + \frac{\pi}{4}\right)$
j. $y = 3\,sec(\pi - x)$

40) Verify each identity.

a. $tan^2\theta sec^2\theta = \frac{sin^2\theta}{cos^4\theta}$

b. $sin^2\theta\, cot\,\theta + \frac{cos^3\theta}{sin\,\theta} = cot\,\theta$

c. $\frac{sec^2\theta}{csc^2\theta} = sec^2\theta - 1$

d. $cos\,\theta\, sin\,\theta + cos^3\theta\, csc\,\theta = cos\,\theta\, csc\,\theta$

41) Solve for x on the interval $[0,2\pi]$.

a. $sin\,2x + cos\,x = 0$

b. $5sin^2x + 12\,sin\,x + 7 = 0$

c. $4cos^2x - 3 = 0$

d. $cos\left(x - \frac{\pi}{2}\right)\left[3\,tan\left(x - \frac{\pi}{2}\right)\right] = 3$

e. $tan^2x - 3 = 0$

f. $\frac{sec\,x}{csc\,x} = 1$

42) Solve for x on the interval $[0, \pi]$.

a. $5\,cos\,x + 3 = 8$

b. $sin\,x\,tan\,x - \sqrt{3}\,sin\,x = 0$

c. $2\,sin\,3x - 1 = 0$

d. $\frac{sec^2x - 1}{sin^2x} = 2$

43) Solve for x. Give all solutions.

a. $sin\,x = \frac{1}{2}$

b. $cos\,x = -\frac{\sqrt{2}}{2}$

c. $tan\,x = \sqrt{3}$

d. $sec\,x = -1$

44) Evaluate each expression.

a. $sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b. $arccos\left(\frac{1}{2}\right)$
c. $tan^{-1}(1)$
d. $arcsin(-1)$
e. $arctan(0)$
f. $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
g. $sin\left[arccos\left(-\frac{3}{5}\right)\right]$
h. $tan\left[sin^{-1}\frac{5}{13}\right]$
i. $csc[tan^{-1}x]$
j. $arccos\left[cos\left(-\frac{\pi}{4}\right)\right]$
k. $sin\left(arcsin\frac{2}{3}\right)$

45) Simplify each expression.

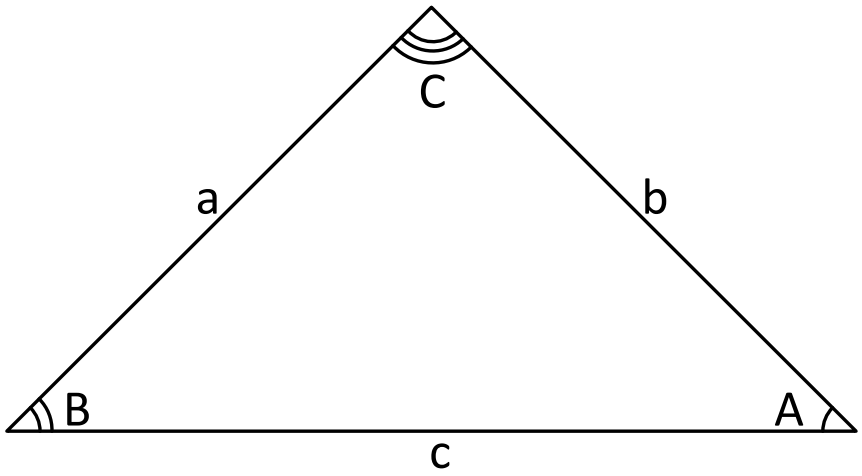
a. $sin(\pi+\theta)$
b. $cos\left(\frac{\pi}{2}-\theta\right)$

46) Find the exact values of $\sin 2u, \cos 2u, \sin\frac{u}{2}, \cos\frac{u}{2}$, given:

a. $\cos u = \frac{4}{5}$ where $\frac{3\pi}{2} < u < 2\pi$
b. $\sin u = \frac{12}{13}$ where $0 < u < \frac{\pi}{2}$
c. $\tan u = -1$ where $\frac{\pi}{2} < u < \pi$
d. $\sin u = -\frac{12}{13}$ where $\pi < u < \frac{3\pi}{2}$

47) Use the power reducing formulas to express $cos^2\theta$ in terms of the first power of cosine.

48) Solve for the missing angles and sides using a calculator. The diagram below is not drawn to scale.




a. $B = 4^{\circ}, A = 25^{\circ}, c = 5$
b. $A = 13^{\circ}, a = 21, b = 13$
c. $C = 42^{\circ}, a = 21, c = 15$
d. $a = 10, b = 20, c = 14$

49) A person casts a shadow that is 8 ft long. The angle of elevation from the tip of the shadow to the person's head is $35^{\circ}$. How tall is the person.

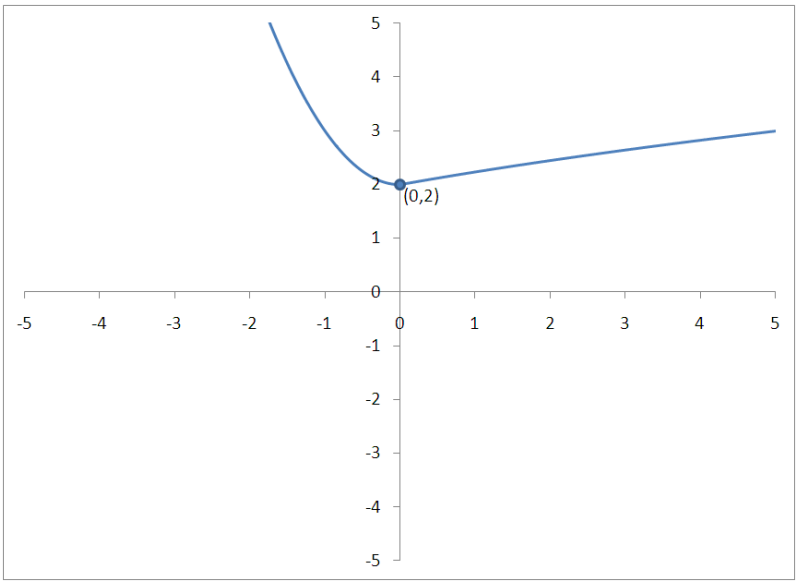
50) A 10 foot ladder is resting against a house. The top of the ladder is 7 feet from the ground. Find the angle the ladder makes with the ground.

51) A tree is growing in a field that is slanted $7^{\circ}$ from the vertical axis. At this moment in time, the tree is leaning towards the sun casting a shadow 50 meters long. The angle between the tip of the shadow and the top of the tree is $27^{\circ}$.

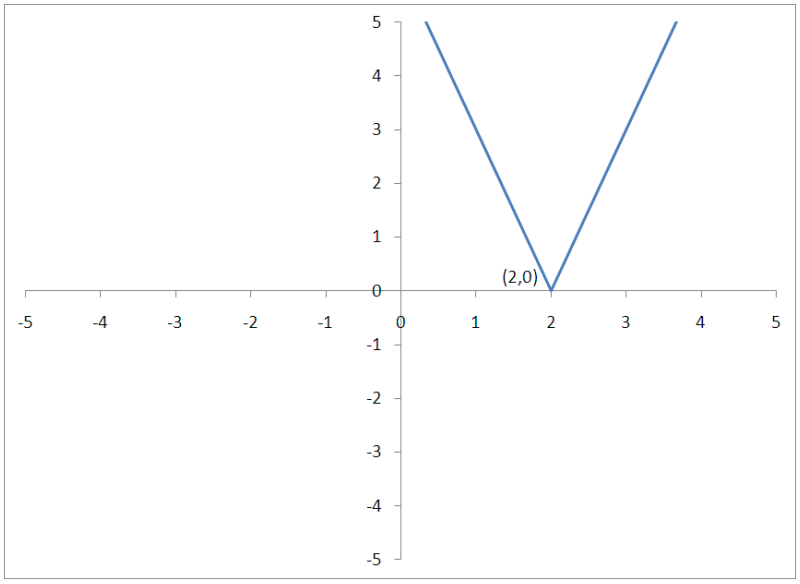
a. Draw a diagram representing the problem.
b. Find the height of the tree.

52) A person is standing 250 feet from the base of a tall building. The angle of elevation to the top of the building is $63^{\circ}$. The angle of elevation to the top of the antenna located on the roof of the building is $68^{\circ}$. How tall is the antenna?

1.
   a. $2x^2(2x^3-1)^2(x^3-4)^{1/3}(13x^3-38)$
   b. $\frac{-(11x+23)}{(x+4)^4(3x+5)^{5/3}}$
   c. $\frac{(7x+2)^{1/2}(91x^2-4x+63)}{2(5x^2+3)^{3/2}}$
2.
   a. $[-1,1) \cup \left[\frac{4}{3}, \infty\right)$
   b. $(-\infty,-5) \cup (3,\infty)$
   c. $\left(\frac{-5}{2}, \frac{1}{2}\right)$
   d. $(-\infty,-5) \cup (-1,3)$
   e. $(-3,1]$
3.
   a. $f(0) = 5$
   b. $f(-2) = -1$
   c. $f(\sqrt{y}) = 2y + 7\sqrt{y} + 5$
   d. $f(2t) = 8t^2 + 14t + 5$
   e. $f(a+3) = 2a^2 + 19a + 44$
   f. $f(-x) = 2x^2 - 7x + 5$
4.
   a. $4x + 2h - 3$
   b. $\frac{-3}{x(x+h)}$
   c. $\frac{1}{\sqrt{x+h-5}+\sqrt{x-5}}$
   d. $\frac{-7}{3x(x+h)}$
5.
   a. $Domain: (-\infty,-5) \cup (-5,7) \cup (7,\infty)$ *or* $\{x|x \neq -5 \text{ and } x \neq 7\}$
   b. $Domain: [-4,4]$ *or* $\{x|-4 \leq x \leq 4\}$
   c. $Domain: (-\infty,-3) \cup (3,\infty)$ *or* $\{x|x < -3 \text{ or } x > 3\}$
   d. $Domain: (-\infty,-2) \cup [9,\infty)$ *or* $\{x|x < -2 \text{ or } x \geq 9\}$
   e. $Domain: [0,4)$ *or* $\{x|0 \leq x < 4\}$
6. $x = -2, 5$

7. 

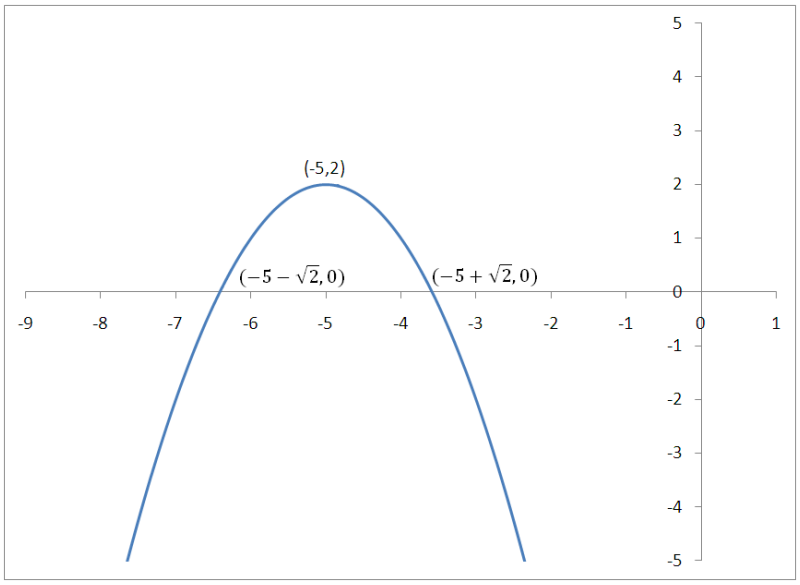

8. $|x+4| = \begin{cases} x+4, & x \geq -4 \\ -x-4, & x < -4 \end{cases}$

9. a. 


$vertical\ stretch; shift\ right\ 2$

Parent function is y=|x|

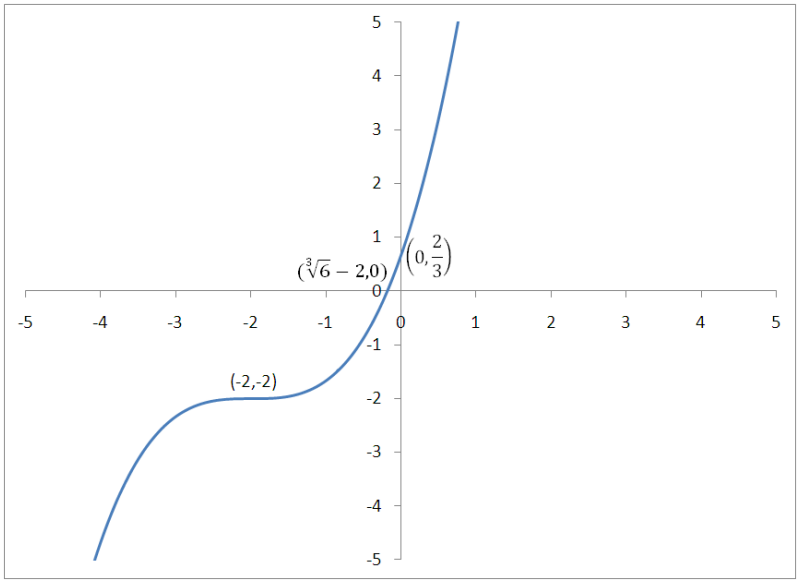
b.




$reflection\ in\ x\ axis; shift\ left\ 5; shift\ up\ 2$

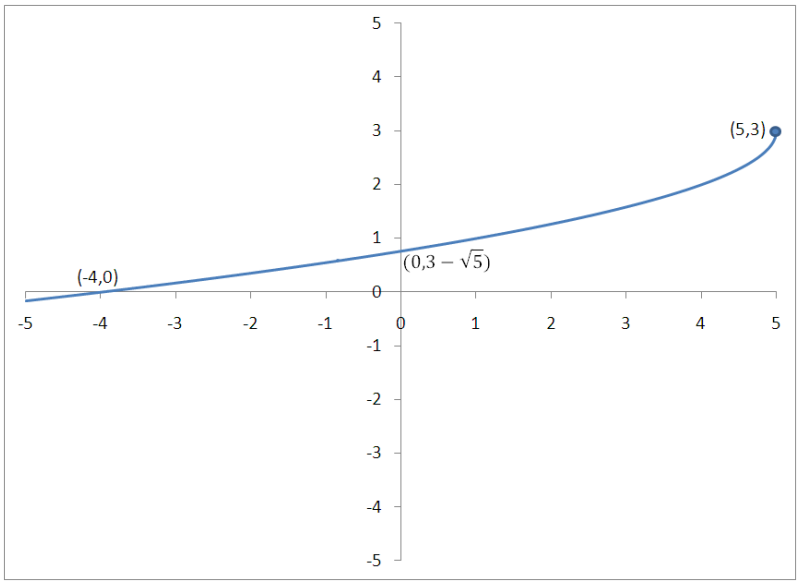
Parent function is $y = x^2$

c.



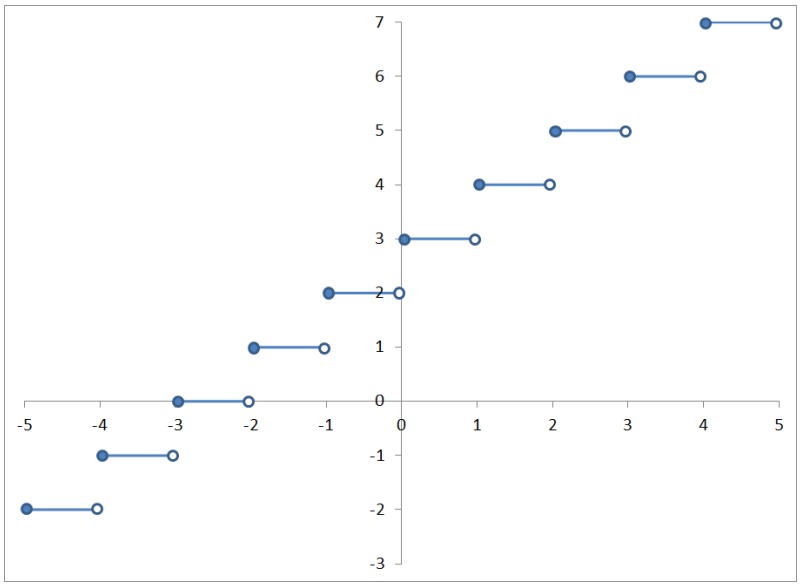

$vertical\ shrink; shift\ left\ 2; shift\ down\ 2$

Parent function is $y = x^3$

d. 


$reflection\ in\ x\ axis; shift\ right\ 5; reflection\ in\ y\ axis; shift\ up\ 3$

Parent function is $y = \sqrt{x}$

e. 

$shift\ up\ 3$

Parent function is $y = [\![x]\!]$

10.

a. $34$

b. $-4$

c. $40\sqrt{2}$

d. $\frac{x^2-5}{\sqrt{x+3}}$ or $\frac{(x^2-5)(\sqrt{x+3})}{x+3}$

e. $Domain: (-3, \infty)\ or\ \{x|x > -3\}$

11.

a. $3\sqrt{3}-5$
b. $\sqrt{x^2-10x+27}$
c. $\sqrt{x^2+4}$

12.

a. $Inverse\ exists$
$f^{-1}(x)=\dfrac{3x+7}{x-1}$
$Domain\ f(x)\text{: }(-\infty,3)\cup(3,\infty)\ \ or\ \{x\,|\,x\neq 3\}$
$Range\ f(x)\text{: }(-\infty,1)\cup(1,\infty)\ \ or\ \ \{y|y\neq 1\}$
$Domain\ f^{-1}(x)\text{: }(-\infty,1)\cup(1,\infty)\ \ or\ \ \{x|x\neq 1\}$
$Range\ f^{-1}(x)\text{: }(-\infty,3)\cup(3,\infty)\ \ or\ \{y\,|\,y\neq 3\}$

b. $Inverse\ exists$
$f^{-1}(x)=x^2+2x+1$
$Domain\ f(x)\text{: }[0,\infty)\ \ or\ \{x\,|\,x\geq 0\}$
$Range\ f(x)\text{: }[-1,\infty)\ or\ \{y|y\geq -1\}$
$Domain\ f^{-1}(x)\text{: }[-1,\infty)\ or\ \{x|x\geq -1\}$
$Range\ f^{-1}(x)\text{: }[0,\infty)\ \ or\ \{y\,|\,y\geq 0\}$

c. $no\ inverse$

d. $no\ inverse$

e. $Inverse\ exists$
$f^{-1}(x)=\dfrac{1}{x}$
$Domain\ f(x)\text{: }(-\infty,0)\cup(0,\infty)\ \ or\ \{x\,|\,x\neq 0\}$
$Range\ f(x)\text{: }\text{: }(-\infty,0)\cup(0,\infty)\ \ or\ \{y\,|\,y\neq 0\}$
$Domain\ f^{-1}(x)\text{: }\text{: }(-\infty,0)\cup(0,\infty)\ \ or\ \{x\,|\,x\neq 0\}$
$Range\ f^{-1}(x)\text{: }(-\infty,0)\cup(0,\infty)\ \ or\ \{y\,|\,y\neq 0\}$

13. $(f\circ g)(x)=f\big(g(x)\big)=f\big(\sqrt[3]{x+7}\big)=(\sqrt[3]{x+7})^3-7=(x+7)-7=x$
$(g\circ f)(x)=\ g\big(f(x)\big)=g(x^3-7)=\ \sqrt[3]{(x^3-7)+7}=\ \sqrt[3]{x^3=x}$

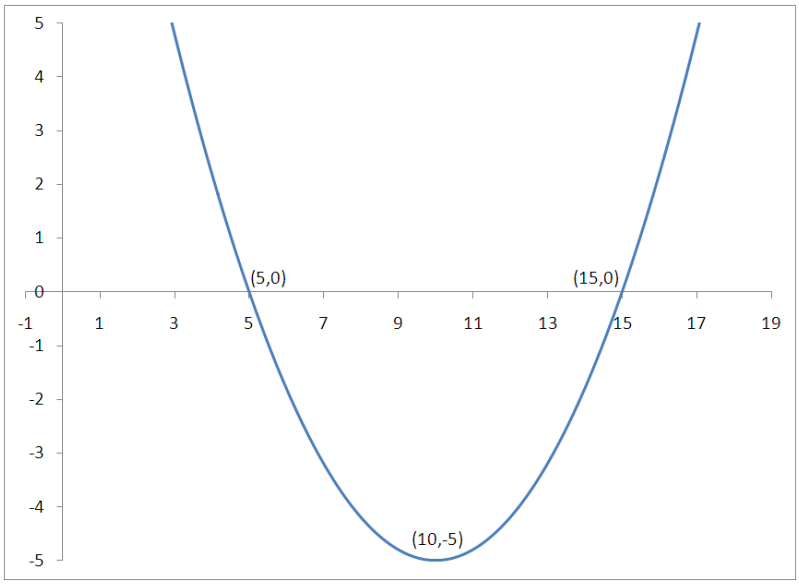
14.

a. $y=3(x-3)^2+2$
b. $y=-3(x-2)^2-3$

15.

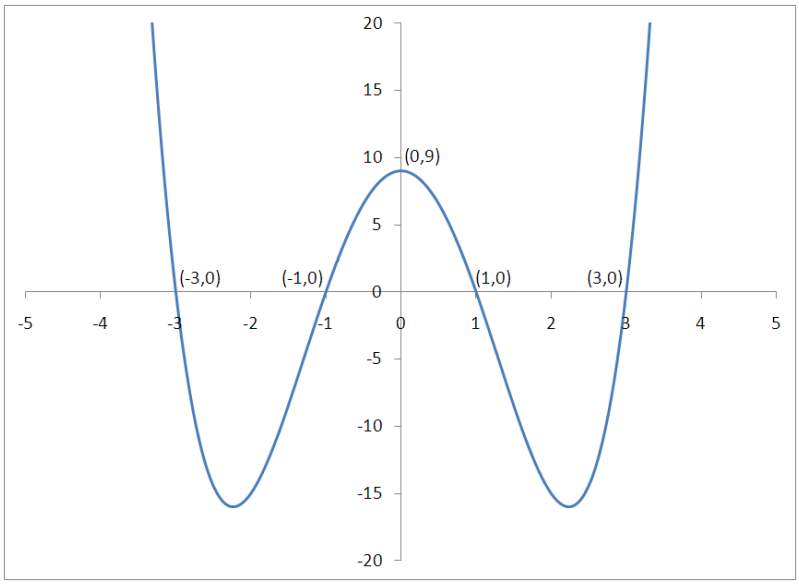
a. $vertex\text{: }(10,-5)$
$axis\ of\ symmetry\text{: }x=10$
$y\ intercept\text{: }(0,15)$
$x\ intercepts\text{: }(5,0)\ and\ (15,0)$

b.

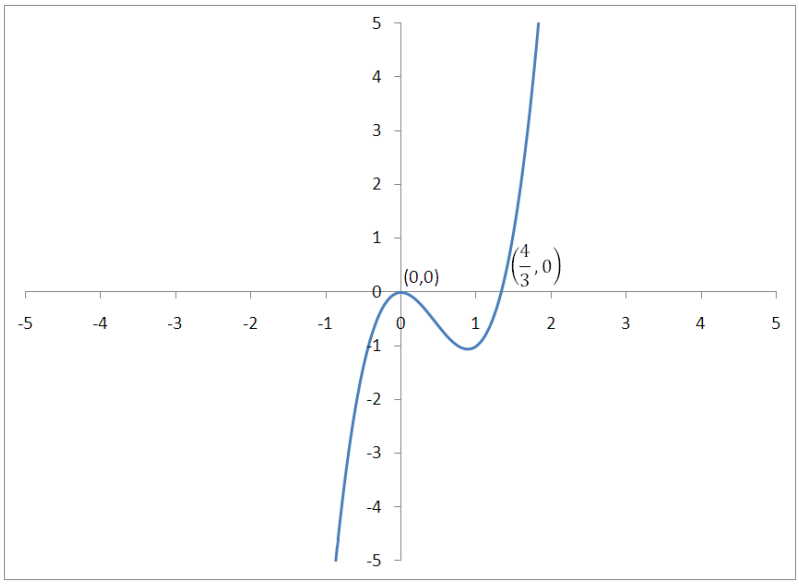



16.

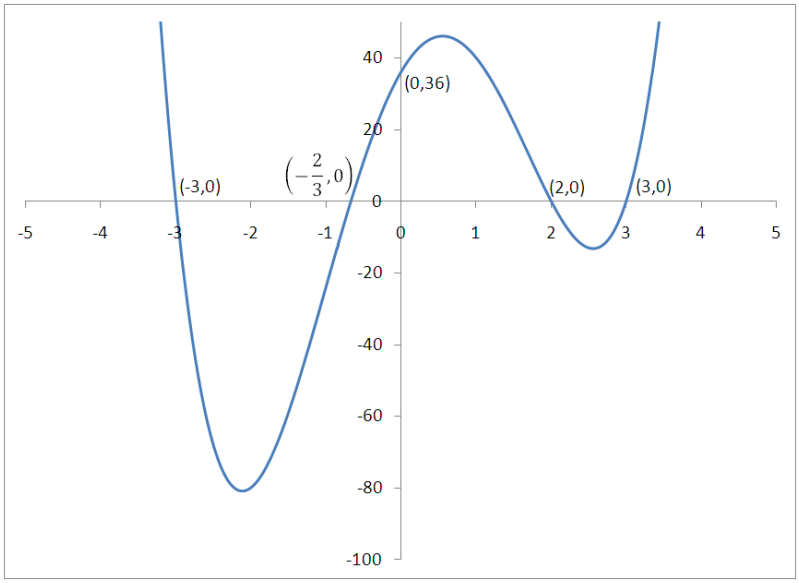
a.



b.




c.




17.

a. $19 + 17i$

b. $\frac{142}{85} - \frac{24}{85}i$

18. $-2 - 4i$

19.

a. $x^2 - 4x - 3 + \frac{26x+4}{x^2+4}$

b. $x^2 + x + 1$

20.

a. $x = 2; multiplicity = 1$
$x = -2; multiplicity = 1$

b. $x = 2; multiplicity = 2$

$x = 0; multiplicity = 1$

21.

a. $x = -2, -1, 2$
b. $x = -3, -2, 2, 3$
c. $x = -2, 2, -2i, 2i$

22. $x = 2, i, -i$

23. $x^3 - x^2 + 9x - 9$

24.

a. $zeros: x = -3, -1, 2$
$f(x) = (x + 1)(x - 2)(x + 3)$
b. $zeros: x = -1, 4, -2i, 2i$
$f(x) = (x + 1)(x - 4)(x - 2i)(x + 2i)$
c. $zeros: x = -5, -2, 0, 7$
$f(x) = x(x + 2)(x + 5)(x - 7)$

25.

a. $Domain: (-\infty, -3) \cup (-3,3) \cup (3, \infty)\ or\ \{x | x \neq -3\ and\ x \neq 3\}$
$vertical\ asymptote: x = -3$
$horizontal\ asymptote: y = 0$
$y\ intercept: \left(0, \frac{1}{3}\right)$
$x\ intercept: none$
b. $Domain: (-\infty, 1) \cup (1, \infty)\ or\ \{x | x \neq 1\}$
$vertical\ asymptote: x = 1$
$slant\ asymptote: y = x + 1$
$y\ intercept: (0,0)$
$x\ intercept: (0,0)$
c. $Domain: All\ Real\ Numbers$
$horizontal\ asymptote: y = 1$
$y\ intercept: (0, -1)$
$x\ intercepts: \left(\frac{-5 + \sqrt{33}}{2}, 0\right)\ and\ \left(\frac{-5 - \sqrt{33}}{2}, 0\right)$

26.

a. $P = 6.4e^{.012t}$
b. $P = 6.4e^{.036} \approx 6.63\ billion$
c. $2024\ and\ 2033$
d. $t = \frac{\ln 2}{.012} \approx 58\ years$

27.

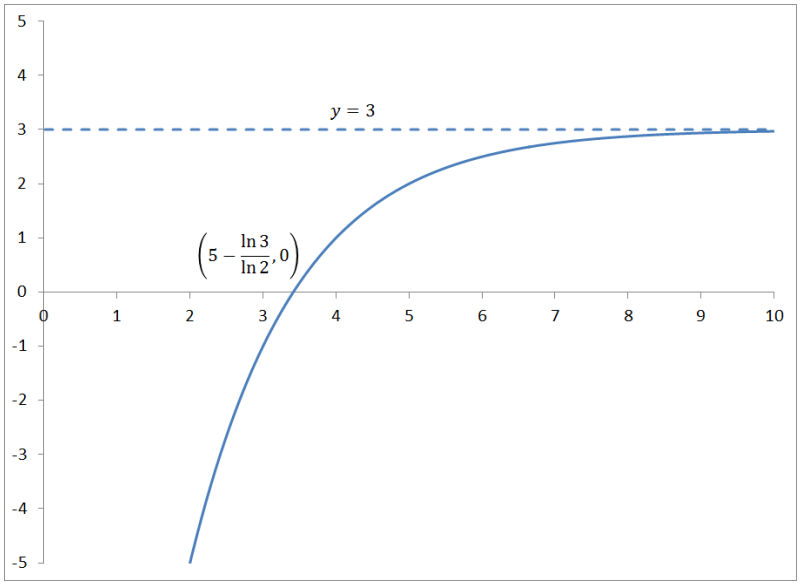
a. $Q = Q_0$
b. $Q = \frac{1}{2}Q_0$
c. $Q = \frac{1}{128}Q_0$

28.

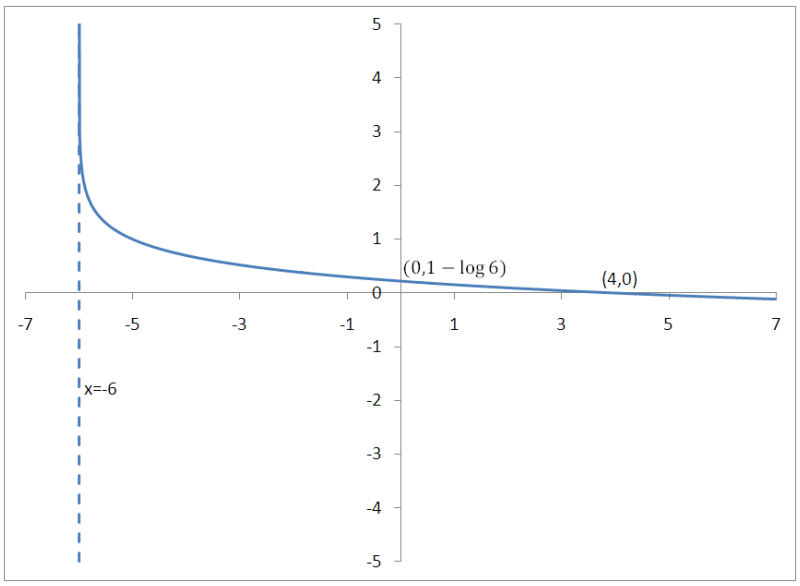
a. $Q = 800e^{-0.000121t}$

b. $Q \approx 785.6\ grams$

29. Graphs

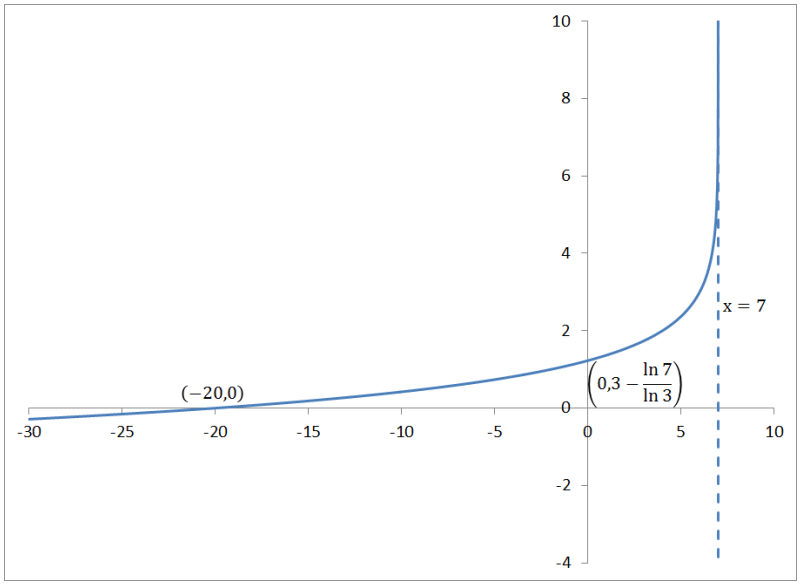
a. 


*Domain*: *All Real Numbers*

b. 

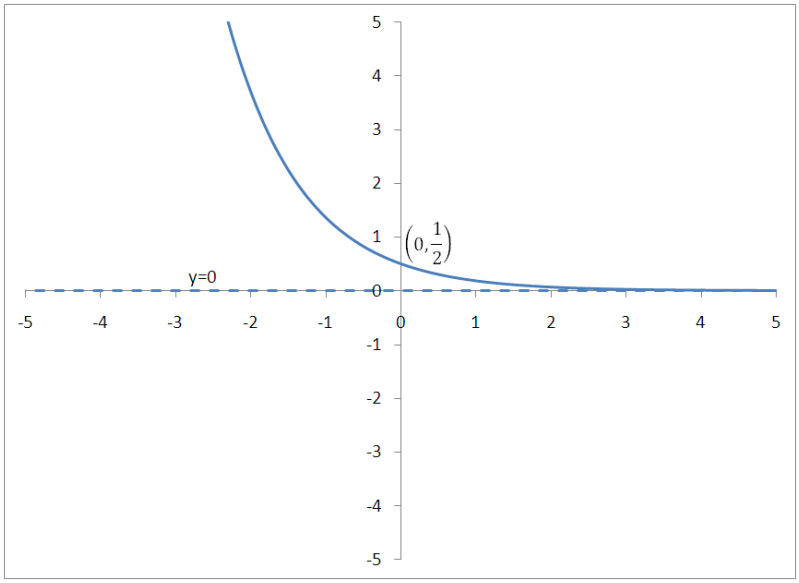

*Domain*: $(-6, \infty)$ *or* $\{x|x > -6\}$

c.

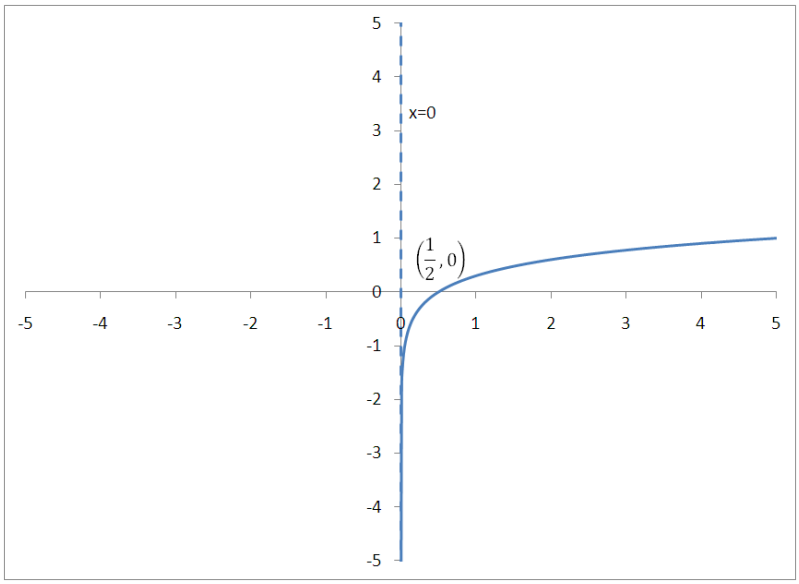



*Domain*: $(-\infty, 7)$ *or* $\{x|x < 7\}$

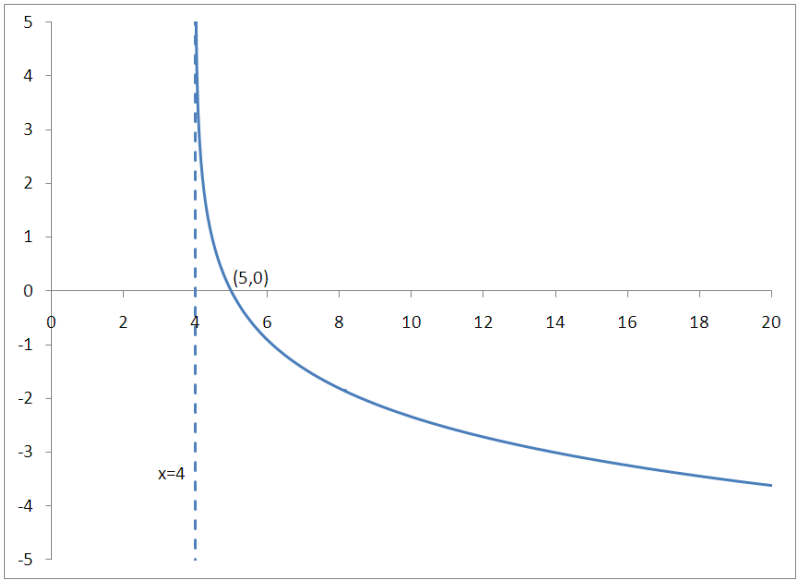
d.




*Domain*: *All Real Numbers*

e. 


$Domain: (0,\infty)\ or\ \{x|x>0\}$

f. 


$Domain: (4,\infty)\ or\ \{x|x>4\}$

30.

a. $\ln\left[\frac{(4x-3)^{1/2}(2x-3)^2}{x^5(x+3)^3}\right]$

b. $\ln\left[\frac{x^4}{(x+5)^2}\right]^{1/3}\ or\ \ln\left[\frac{x^{4/3}}{(x+5)^{2/3}}\right]$

c. $\log\left[\frac{3}{y^{1/2}z}\right]$

31.

a. $\frac{1}{2}\log 2+\frac{3}{2}\log x-\log y-2\log z$

b. $\frac{1}{3}[\ln(x-5)+\ln y-\ln 3-\ln z]$

c. $3[\ln(5x+z)-\ln x-\ln y]$

32.

a. $x=2$
b. $x=\frac{1}{3}$
c. $x=100$
d. $x=3$
e. $x=-5$
f. $x=\frac{11}{3}$
g. $x=1$
h. $x=500$
i. $x=\frac{1}{2}$
j. $x=-10,10$
k. $x=1$
l. $x=e^3$
m. $x=6$
n. $no\ solution$
o. $x=\ln 6$
p. $x=-3,2$

33.

a. $A\approx \$30{,}648.54$
b. $t\approx 4.33\ years$

34.

a. $V=-2600t+25000$
b. $V=25000e^{-0.147t}$

35.

a. $\sin\theta=0;\ \cos\theta=1;\ \tan\theta=0;\ \csc\theta=undefined;\ \sec\theta=1;\ \cot\theta=undefined$
b. $\sin\theta=\frac{\sqrt{2}}{2};\ \cos\theta=-\frac{\sqrt{2}}{2};\ \tan\theta=-1;\ \csc\theta=\sqrt{2};\ \sec\theta=-\sqrt{2};\ \cot\theta=-1$
c. $\sin\theta=\frac{\sqrt{3}}{2};\ \cos\theta=-\frac{1}{2};\ \tan\theta=-\sqrt{3};\ \csc\theta=\frac{2\sqrt{3}}{3};\ \sec\theta=-2;\ \cot\theta=-\frac{\sqrt{3}}{3}$
d. $\sin\theta=0;\ \cos\theta=-1;\ \tan\theta=0;\ \csc\theta=undefined;\ \sec\theta=-1;\ \cot\theta=undefined$
e. $\sin\theta=-\frac{\sqrt{3}}{2};\ \cos\theta=\frac{1}{2};\ \tan\theta=-\sqrt{3};\ \csc\theta=-\frac{2\sqrt{3}}{3};\ \sec\theta=2;\ \cot\theta=-\frac{\sqrt{3}}{3}$
f. $\sin\theta=-\frac{1}{2};\ \cos\theta=-\frac{\sqrt{3}}{2};\ \tan\theta=\frac{\sqrt{3}}{3};\ \csc\theta=-2;\ \sec\theta=-\frac{2\sqrt{3}}{3};\ \cot\theta=\sqrt{3}$
g. $\sin\theta=\frac{\sqrt{2}}{2};\ \cos\theta=-\frac{\sqrt{2}}{2};\ \tan\theta=-1;\ \csc\theta=\sqrt{2};\ \sec\theta=-\sqrt{2};\ \cot\theta=-1$
h. $\sin\theta=-\frac{1}{2};\ \cos\theta=\frac{\sqrt{3}}{2};\ \tan\theta=-\frac{\sqrt{3}}{3};\ \csc\theta=-2;\ \sec\theta=\frac{2\sqrt{3}}{3};\ \cot\theta=-\sqrt{3}$
i. $\sin\theta=1;\ \cos\theta=0;\ \tan\theta=undefined;\ \csc\theta=1;\ \sec\theta=undefined;\ \cot\theta=0$

36.

a. $\sin\theta=\frac{4}{5};\ \cos\theta=\frac{3}{5};\ \tan\theta=\frac{4}{3};\ \csc\theta=\frac{5}{4};\ \sec\theta=\frac{5}{3};\ \cot\theta=\frac{3}{4}$
b. $\sin\theta=\frac{9\sqrt{181}}{181};\ \cos\theta=\frac{10\sqrt{181}}{181};\ \tan\theta=\frac{9}{10};\ \csc\theta=\frac{\sqrt{181}}{9};\ \sec\theta=\frac{\sqrt{181}}{10};\ \cot\theta=\frac{10}{9}$

37.

a. $\sin\theta = -\frac{\sqrt{2}}{2};\ \cos\theta = -\frac{\sqrt{2}}{2};\ \tan\theta = 1;\ \csc\theta = -\sqrt{2};\ \sec\theta = -\sqrt{2};\ \cot\theta = 1$

b. $\sin\theta = \frac{1}{2};\ \cos\theta = -\frac{\sqrt{3}}{2};\ \tan\theta = -\frac{\sqrt{3}}{3};\ \csc\theta = 2;\ \sec\theta = -\frac{2\sqrt{3}}{3};\ \cot\theta = -\sqrt{3}$

c. $\sin\theta = \frac{\sqrt{21}}{5};\ \cos\theta = -\frac{2}{5};\ \tan\theta = -\frac{\sqrt{21}}{2};\ \csc\theta = \frac{5\sqrt{21}}{21};\ \sec\theta = -\frac{5}{2};\ \cot\theta = -\frac{2\sqrt{21}}{21}$

d. $\sin\theta = \frac{1}{2};\ \cos\theta = \frac{\sqrt{3}}{2};\ \tan\theta = \frac{\sqrt{3}}{3};\ \csc\theta = 2;\ \sec\theta = \frac{2\sqrt{3}}{3};\ \cot\theta = \sqrt{3}$

38.

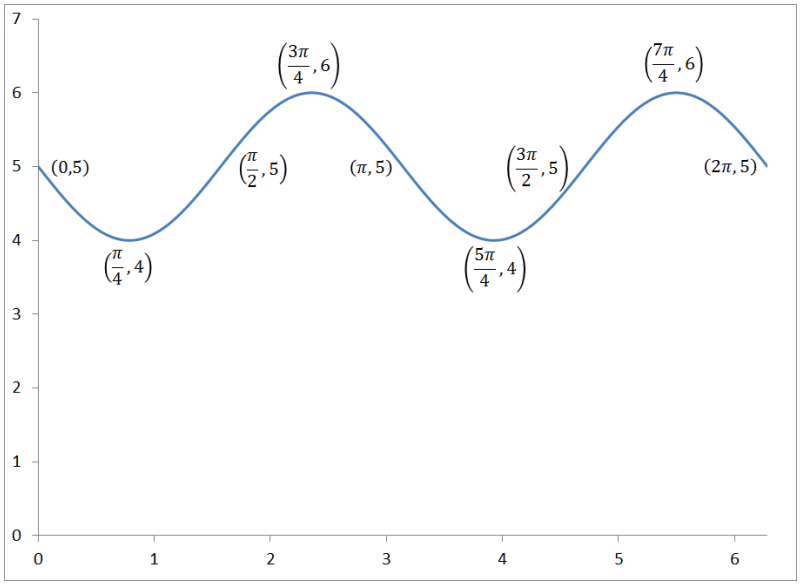
a. $Amplitude = 5; Period = \pi; Phase\ Shift = 0$

b. $Amplitude = \pi; Period = 8; Phase\ Shift = 0$

c. $Amplitude = 3; Period = \pi; Phase\ Shift = +\frac{\pi}{2}$

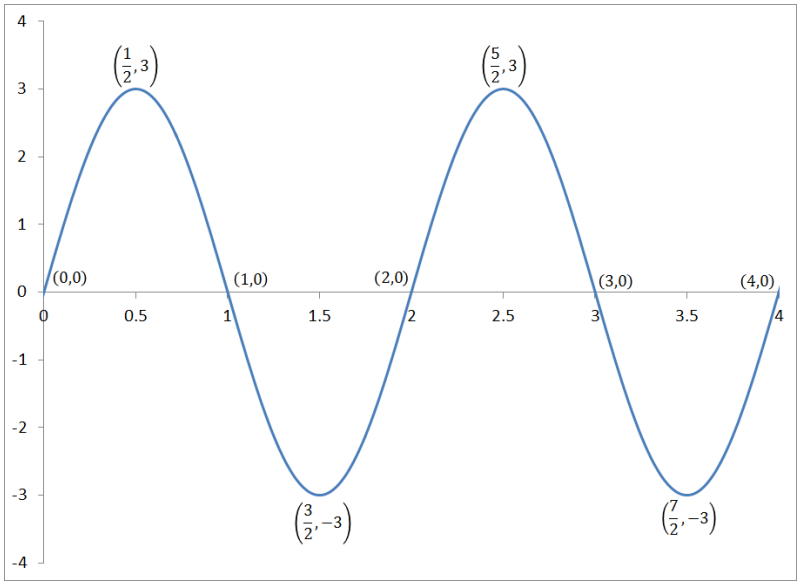
39.

a.



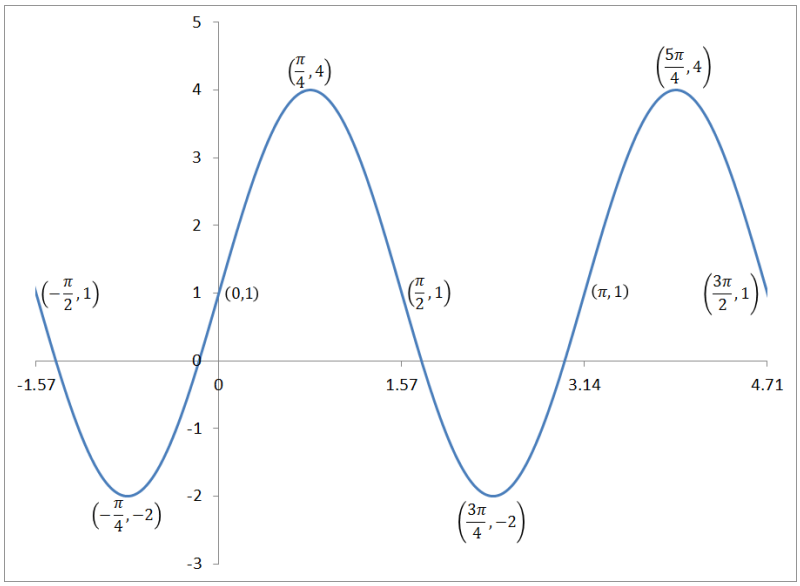

*Domain: All Real Numbers*

b.



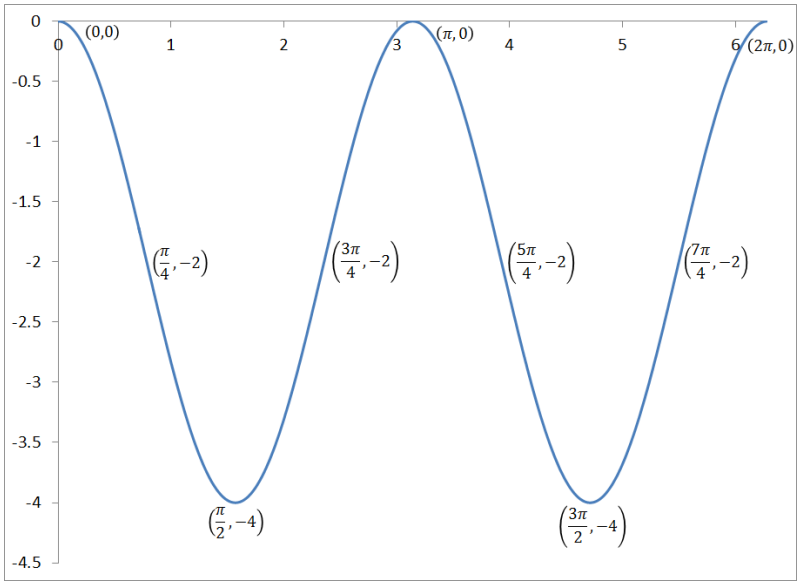

*Domain*: *All Real Numbers*

c.



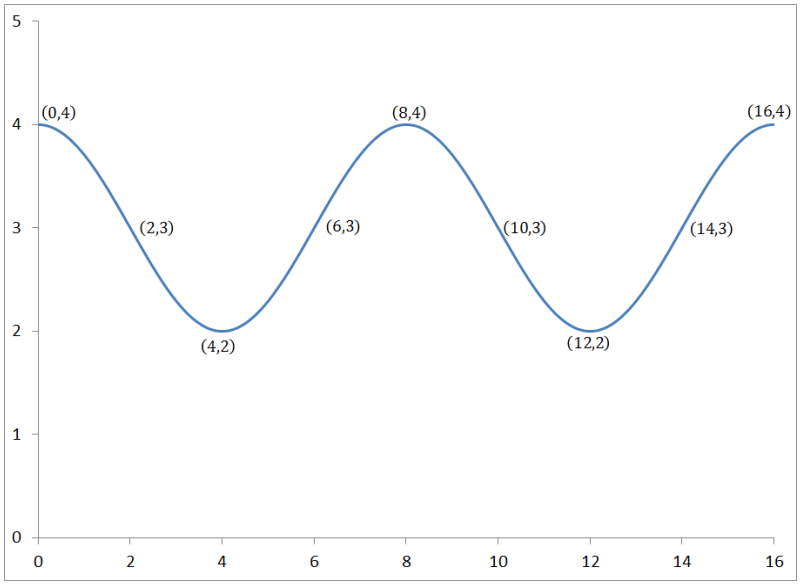

*Domain*: *All Real Numbers*

d.



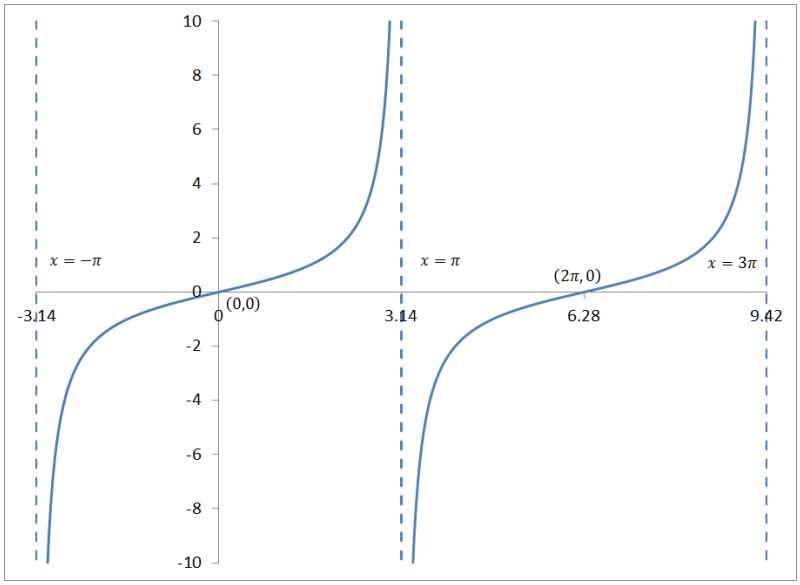

*Domain: All Real Numbers*

e.



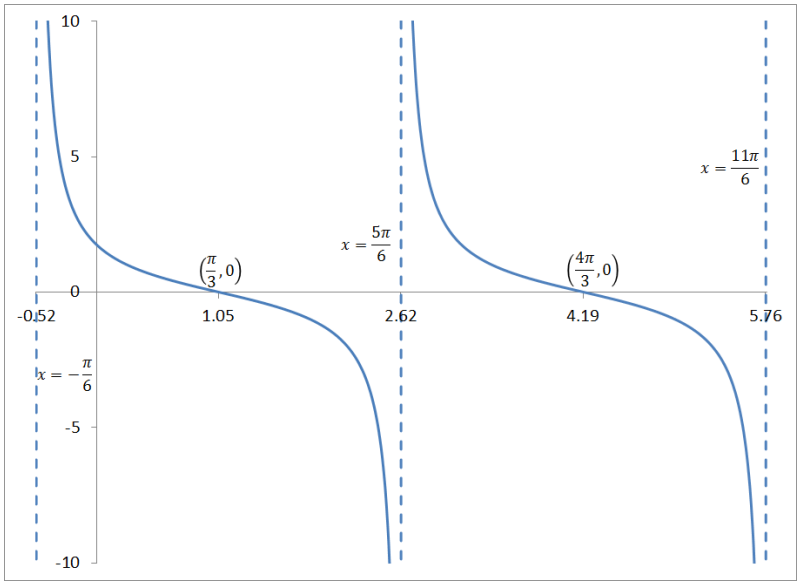

*Domain: All Real Numbers*

f.



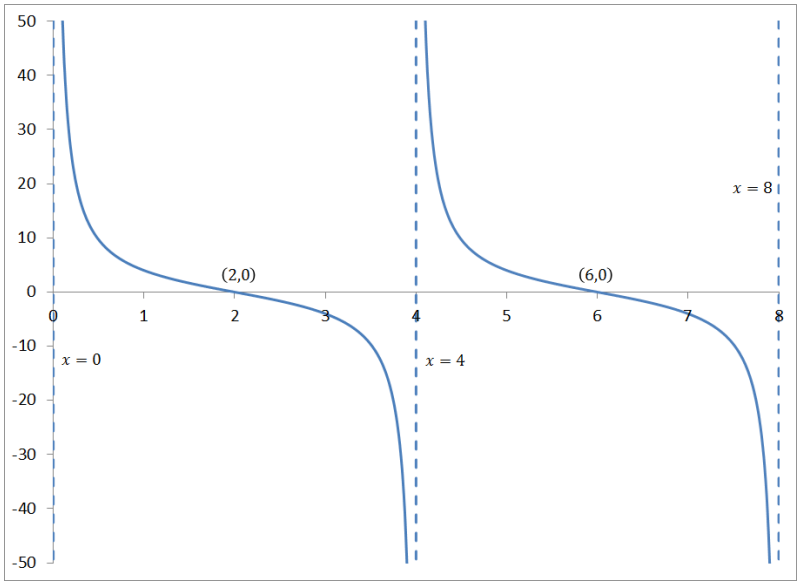

$Domain: \{x | x \neq n\pi \text{ where } n \text{ is an odd integer}\}$

g.



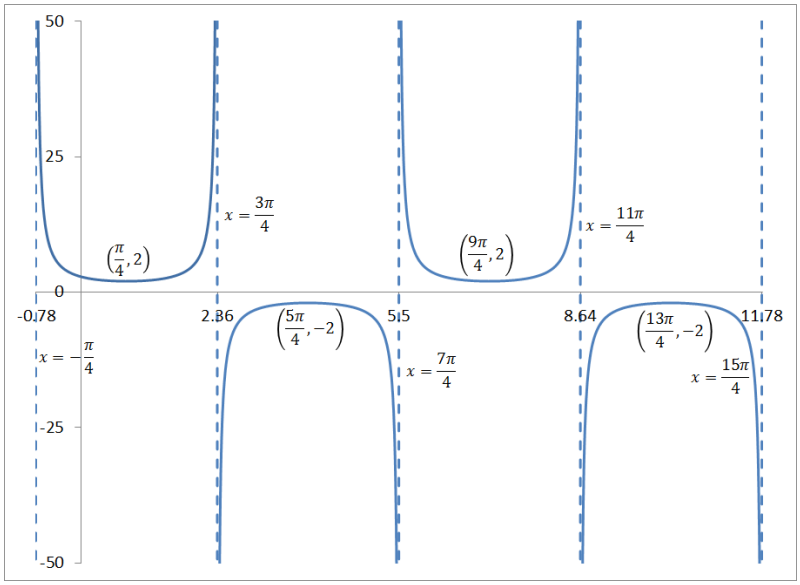

$Domain: \left\{x \middle| x \neq \frac{\pi(3n+2)}{6} \text{ where } n \text{ is an odd integer}\right\}$

h.

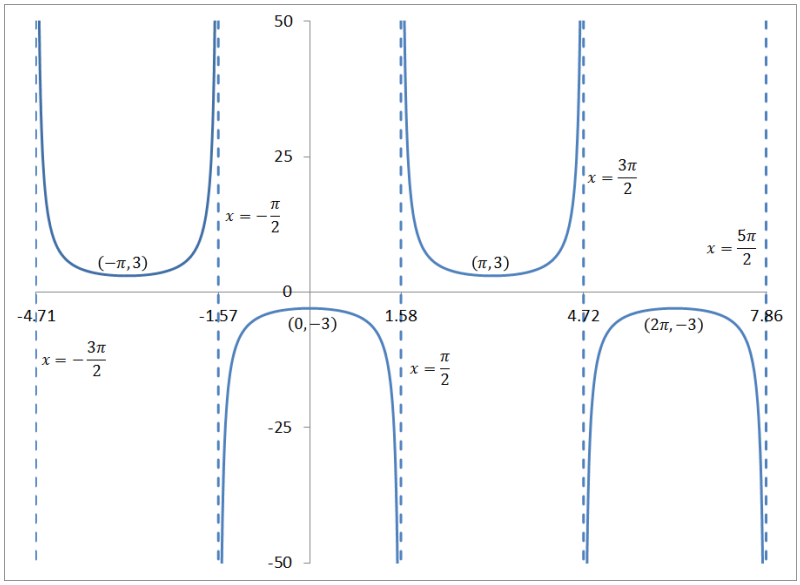



$Domain: \{x | x \neq 4n \text{ where } n \text{ is an integer}\}$

i.




$Domain: \left\{x \middle| x \neq \frac{\pi(4n-1)}{4} \text{ where } n \text{ is an integer}\right\}$

j. 


$Domain: \{x|x \neq \frac{n\pi}{2}\ where\ n\ is\ an\ odd\ integer\}$

40.

a. $\tan^2\theta \sec^2\theta = \left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{1}{\cos^2\theta}\right) = \frac{\sin^2\theta}{\cos^4\theta}$

b. $\sin^2\theta \cot\theta + \frac{\cos^3\theta}{\sin\theta} = \sin^2\theta\left(\frac{\cos\theta}{\sin\theta}\right) + \frac{\cos^3\theta}{\sin\theta} = \frac{\sin^2\theta\cos\theta}{\sin\theta} + \frac{\cos^3\theta}{\sin\theta} = \frac{\sin^2\theta\cos\theta+\cos^3\theta}{\sin\theta} = \frac{\cos\theta(\sin^2\theta+\cos^2\theta)}{\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

c. $\frac{\sec^2\theta}{\csc^2\theta} = \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta = \sec^2\theta - 1$

d. $\cos\theta\sin\theta + \cos^3\theta\csc\theta = \cos\theta\sin\theta + \frac{\cos^3\theta}{\sin\theta} = \frac{\cos\theta\sin^2\theta}{\sin\theta} + \frac{\cos^3\theta}{\sin\theta} = \frac{\cos\theta\sin^2\theta+\cos^3\theta}{\sin\theta} = \frac{\cos\theta(\sin^2\theta+\cos^2\theta)}{\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cos\theta\csc\theta$

41.

a. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

b. $x = \frac{3\pi}{2}$

c. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

d. $x = no\ solution$

e. $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

f. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

42.

a. $x = 0$

b. $x = 0, \frac{\pi}{3}, \pi$

c. $x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$

d. $x = \frac{\pi}{4}, \frac{3\pi}{4}$

43.

a. $x = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi \; where \; n \; is \; an \; integer$

b. $x = \frac{3\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi \; where \; n \; is \; an \; integer$

c. $x = \frac{\pi}{3} + n\pi \; where \; n \; is \; an \; integer$

d. $x = n\pi \; where \; n \; is \; an \; odd \; integer$

44.

a. $\frac{\pi}{3}$

b. $\frac{\pi}{3}$

c. $\frac{\pi}{4}$

d. $-\frac{\pi}{2}$

e. $0$

f. $\frac{5\pi}{6}$

g. $\frac{4}{5}$

h. $\frac{5}{12}$

i. $\frac{\sqrt{x^2+1}}{x}$

j. $\frac{\pi}{4}$

k. $\frac{2}{3}$

45.

a. $-\sin\theta$

b. $\sin\theta$

46.

a. $\sin 2u = -\frac{24}{25}; \cos 2u = \frac{7}{25}; \; \sin\frac{u}{2} = \frac{\sqrt{10}}{10}; \cos\frac{u}{2} = -\frac{3\sqrt{10}}{10}$

b. $\sin 2u = \frac{120}{169}; \cos 2u = -\frac{119}{169}; \; \sin\frac{u}{2} = \frac{2\sqrt{13}}{13}; \cos\frac{u}{2} = \frac{3\sqrt{13}}{13}$

c. $\sin 2u = -1; \cos 2u = 0; \; \sin\frac{u}{2} = \frac{\sqrt{2+\sqrt{2}}}{2}; \cos\frac{u}{2} = \frac{\sqrt{2-\sqrt{2}}}{2}$

d. $\sin 2u = \frac{120}{169}; \cos 2u = -\frac{119}{169}; \; \sin\frac{u}{2} = \frac{3\sqrt{13}}{13}; \cos\frac{u}{2} = -\frac{2\sqrt{13}}{13}$

47. $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

48. Answers may vary in last decimal place due to rounding errors

a. $C = 151°; \; a = 4.36; \; b = 0.72$

b. $B = 8.00°; \; C = 159.00°; \; c = 33.46$

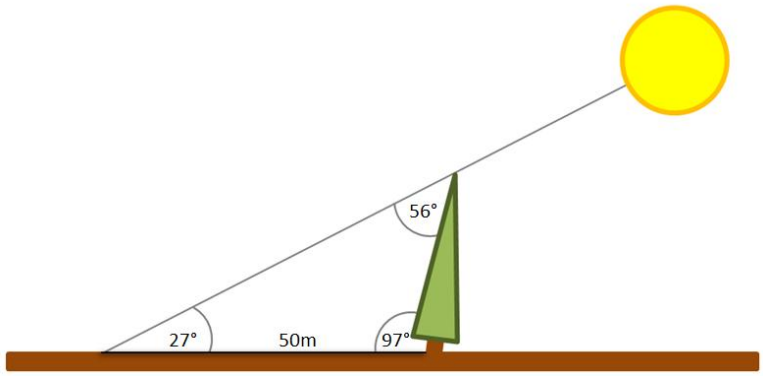
c. $A = 69.52°; \; B = 68.48°; \; b = 20.85 \; or \; A = 110.48° \; B = 27.52° \; b = 10.36$

d. $A = 27.66°; \; B = 111.80°; \; C = 40.54°$

49. $5.6ft$

50. $44.4°$

51.




a.

b. $27.4m$

52. $128.1ft$